Physics 503 **Homework Set 4** **Fall 2016**

Zangwill 4.25 Analyze This Potential

An asymptotic (long-distance) electrostatic potential has the form

$$\phi(r,\theta,\varphi) = Ar + \frac{Bx^2}{r^5} + \text{higher-order terms.}$$

(a) Use a traceless Cartesian multipole expansion to show that no localized charge distribution exists which can produce an asymptotic potential of this form.

(b) Repeat part (a) using a primitive Cartesian multipole expansion.

(c) Show that a suitable $\rho(r)$ can be found which produces the given potential if we remove the restriction that the charge distribution is localized.

Zangwill 5.5 The Charge Distribution Induced on a Neutral Sphere

A point charge q lies a distance $r > R$ from the center of an uncharged, conducting sphere of radius R. Express the induced surface charge density in the form

$$\sigma(\theta) = \sum_{l=1}^{\infty} \sigma_l P_l(\cos\theta)$$

where θ is the polar angle measured from a positive z-axis which points from the sphere center to the point charge.

(a) Show that the total electrostatic energy is

$$U_E = \frac{1}{\epsilon_0}\sum_{l=1}^{\infty} \frac{\sigma_l}{2l+1}\left[\frac{R^3\sigma_l}{2}\frac{4\pi}{2l+1} + \frac{qR^{l+2}}{r^{l+1}}\right].$$

(b) Use Thomson's theorem to find $\sigma(\theta)$.

Zangwill 5.10 Charge Induction by a Dipole

A point dipole $\mathbf{p}$ is placed at $\mathbf{r} = \mathbf{r}_0$ outside a grounded conducting sphere of radius R. Use Green's reciprocity (and a comparison system with zero volume charge density) to find the charge drawn up from ground onto the sphere.