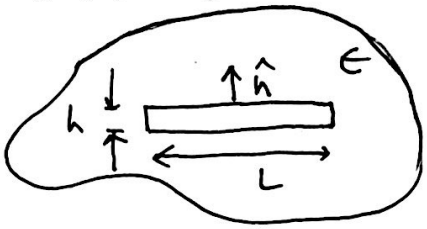
Zangwill 6.5 The Field at the Center of a Polarized Cube A cube is polarized uniformly parallel to one of its edges. Show that the electric field at the center of the cube is $\mathbf{E}(0) = -\mathbf{P}/3\epsilon_0$. Compare with $\mathbf{E}(0)$ for a uniformly polarized sphere. Hint: Recall the definition of solid angle.

Zangwill 6.11 Cavity Field A uniform electric field $\mathbf{E}_0$ exists throughout a homogeneous dielectric with permittivity ϵ. What is the electric field inside a vacuum cavity cut out of the interior of the dielectric in the shape of a rectangular pancake with dimensions $L \times L \times h$? Assume that $h \ll L$ and express $\mathbf{E}_{\text{cav}}$ entirely in terms of $\mathbf{E}_0$, $\hat{\mathbf{n}}$, ϵ, and ϵ_0.



Zangwill 6.15 A Parallel-Plate Capacitor with an Air Gap An air-gap capacitor with parallel-plate area A discharges by the electrical breakdown of the air between its parallel plates (separation d) when the voltage between its plates exceeds V_0. Lay a slab with dielectric constant κ and thickness $t < d$ on the surface of the lower plate and maintain a potential difference V between the plates.

(a) Find the capacitance of this structure with the dielectric slab present.

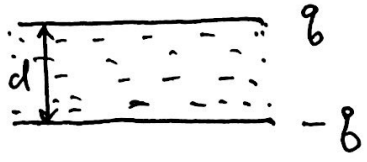
(b) Show that the value of V where electric breakdown occurs in the air portion of the capacitor gap is

$$V' = V_0 \left[1 - \frac{t}{d}\left(1 - \frac{1}{\kappa}\right)\right].$$

Zangwill 6.18 Surface Polarization Charge Point charges $q_1, q_2, \ldots, q_N$ are embedded in a body with permittivity κ_{in}. The latter is itself embedded in a body with permittivity κ_{out}. Find the total polarization charge Q_{pol} induced on the boundary between the two dielectrics.

Zangwill 6.19 An Elastic Dielectric The parallel-plate capacitor shown below is made of two identical conducting plates of area A carrying charges $\pm q$. The capacitor is filled with a compressible dielectric solid with permittivity and elastic energy

$$U_e = \frac{1}{2}k(d - d_0)^2.$$



(a) Find the equilibrium separation between the plates $d(q)$.

(b) Sketch the potential difference between the plates $V(q)$. Comment on any unusual behavior of the differential capacitance $C_d(q) = dq/dV$.

Zangwill 6.23 Two Dielectric Interfaces The figure shows two fixed-potential capacitors filled with equal amounts of two different types of simple dielectric matter. Use the stress tensor method to compare the force per unit area which acts on the two dielectric interfaces. Express your answer in terms of the electric field E_0 which would be present if the dielectric matter were absent.