Exercise 31.3

Description: The voltage across the terminals of an ac power supply varies with time according to \( V = V_0 \cos(\omega t) \). The voltage amplitude is \( V_0 \). (a) What is the root-mean-square potential difference \( V_{\text{rms}} \)? (b) What is the average potential difference \( V_{\text{avg}} \) between the two terminals of the power supply?

The voltage across the terminals of an ac power supply varies with time according to \( V = V_0 \cos(\omega t) \). The voltage amplitude is \( V_0 = 41.0 \, \text{V} \).

Part A

What is the root-mean-square potential difference \( V_{\text{rms}} \)?

ANSWER:

\[
V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 29.0 \, \text{V}
\]

Part B

What is the average potential difference \( V_{\text{avg}} \) between the two terminals of the power supply?

ANSWER:

\[
V_{\text{avg}} = 0 \, \text{V}
\]

Phasors Explained

Description: Introduces phasors with questions both general and in the context of AC circuits. Recommended after resistance, inductance, and capacitance in AC circuits have been discussed. Includes a part on series LCR circuits and another part on parallel LCR circuits.

Learning Goal:

To understand the concept of phasor diagrams and be able to use them to analyze AC circuits (those with sinusoidally varying current and voltage).

Phasor diagrams provide a convenient graphical way of representing the quantities that change with time along with \( \cos(\omega t + \phi) \), which makes such diagrams useful for analyzing AC circuits with their inherent phase shifts between voltage and current. You have studied the behavior of an isolated resistor, inductor, and capacitor connected to an AC source. However, when a circuit contains more than one element (for instance, a resistor and a capacitor or a resistor and an inductor or all three elements), phasors become a useful tool that allows us to calculate currents and voltages rather easily and also to visualize some important processes taking place in the AC circuit, such as resonance.

Let us assume that a certain quantity \( I(t) \) changes over time as \( I(t) = I_0 \cos(\omega t) \). A phasor is a vector whose length represents the amplitude \( I_0 \) (see the diagram). This vector is assumed to rotate counterclockwise with angular frequency \( \omega \); that way, the horizontal component of the vector represents the actual value \( I(t) \) at any given moment.

In this problem, you will answer some basic questions about phasors and prepare to use them in the analysis of various AC circuits.
In parts A - C consider the four phasors shown in the diagram. Assume that all four phasors have the same angular frequency \( \omega \).

**Part A**

At the moment \( t \) depicted in the diagram, which of the following statements is true?

**ANSWER:**

- \( I_2 \) leads \( I_1 \) by \( \pi \).
- \( I_1 \) leads \( I_2 \) by \( \pi \).
- \( I_2 \) leads \( I_1 \) by \( \frac{\pi}{2} \).
- \( I_1 \) leads \( I_2 \) by \( \frac{\pi}{2} \).

**Part B**

At the moment shown in the diagram, which of the following statements is true?

**ANSWER:**

- \( I_2 \) lags \( I_3 \) by \( \pi \).
- \( I_3 \) lags \( I_2 \) by \( 2\pi \).
- \( I_2 \) lags \( I_3 \) by \( \frac{\pi}{2} \).
- \( I_3 \) lags \( I_2 \) by \( \frac{\pi}{2} \).

**Part C**

At the moment shown in the diagram, which of the following statements is true?

**ANSWER:**
Let us now consider some basic applications of phasors to AC circuits.

- For a resistor, the current and the voltage are always in phase.
- For an inductor, the current lags the voltage by $\frac{\pi}{2}$.
- For a capacitor, the current leads the voltage by $\frac{\pi}{2}$.

**Part D**

Consider this diagram. Let us assume that it describes a series circuit containing a resistor, a capacitor, and an inductor. The current in the circuit has amplitude $I_2$, as indicated in the figure.

Which of the following choices gives the correct respective labels of the voltages across the resistor, the capacitor, and the inductor?

**ANSWER:**

- $V_1; V_2; V_3$
- $V_1; V_2; V_4$
- $V_1; V_4; V_2$
- $V_3; V_2; V_4$
- $V_3; V_4; V_2$

**Part E**

Now consider a diagram describing a parallel AC circuit containing a resistor, a capacitor, and an inductor. This time, the voltage across each of these elements of the circuit is the same; on the diagram, it is represented by the vector labeled $V_0$.

The currents in the resistor, the capacitor, and the inductor are represented respectively by which vectors?

**ANSWER:**
\[ i = I \cos(\omega t), \text{ where } I \text{ is the maximum current and } \omega = 2\pi f \text{ (f is the frequency of the current source).} \]

The relationship between the current and voltage in an ac circuit works according to Ohm's law. Consider just the resistor in the circuit. Because the current changes in time, the voltage across the resistor also changes. However, both \( V_R \) and \( i \) will be at a maximum at the same time. The maximum voltage across the resistor is given by

\[ V_R = IR. \]

Recall that an inductor is designed to oppose any change in current in the circuit. Although an inductor has no resistance, there is a potential difference \( V_L \) across the ends of the inductor. Unlike the case for the resistor, \( i \) and \( V_L \) do not reach maximum values at the same time. The voltage reaches a maximum before the current.

The maximum potential difference across the inductor is

\[ V_L = I\omega L. \]

By defining the quantity \( \omega L \) as the inductive reactance \( X_L \), \( V_L \) can be rewritten as \( V_L = IX_L \). This equation is similar to Ohm's law.

**Part A**

An \( L-R-C \) circuit, operating at 60 Hz, has an inductor with an inductance of \( 1.53 \times 10^{-3} \) H, a capacitance of \( 1.67 \times 10^{-2} \) F, and a resistance of 0.329 \( \Omega \). What is the inductive reactance of this circuit?

Enter your answer numerically in ohms.

**Hint 1.** Find \( \omega \)

The frequency \( f \) is 60 Hz. You must first find \( \omega \) from the frequency using \( \omega = 2\pi f \). What is \( \omega \)?

Enter your answer numerically in inverse seconds.

ANSWER:

\[ \omega = 377 \ s^{-1} \]
Because $X_L$ is proportional to the frequency, low-frequency ac currents pass through an inductor more easily than high-frequency ac currents. Hence circuits containing inductors are often used as filters that allow low frequencies but not high frequencies to pass. These filters are called low-pass filters.

A capacitor is designed to store energy by allowing charge to build up on its plates. Although a capacitor has no resistance in an ac circuit, there is a potential difference $v_C$ across the plates of the capacitor. The maximum values of $i$ and $v_C$ do not occur at the same time. The voltage reaches a maximum after the current.

The maximum potential difference across the inductor is

$$V_C = \frac{I}{\omega C}.$$  

By defining the quantity $1/\omega C$ as the capacitive reactance $X_C$, $V_C$ can be rewritten as $V_C = IX_C$. As with the case for the inductor, this equation is similar to Ohm's law.

**Part B**

What is the capacitive reactance of the circuit in Part A?

**Enter your answer numerically in ohms.**

**ANSWER:**

$$X_C = 0.159 \ \Omega$$

Because $X_C$ is inversely proportional to the frequency, high-frequency ac currents pass through a capacitor more easily than low-frequency ac currents. Hence circuits containing capacitors are often used as filters that allow high frequencies but not low frequencies to pass. These filters are called high-pass filters.

The capacitive reactance, the inductive reactance, and the resistance of the circuit can be combined to give the impedance $Z$ of the circuit. The impedance is a measure of the total reactance and resistance of the circuit and is similar to the equivalent resistance that can be found from various resistors in a dc circuit. Because $v_L$ and $v_C$ do not reach maximum values at the same time as $i$, the impedance is not found by adding $R$, $X_L$, and $X_C$. Instead, the impedance is found using

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$  

The analogy to Ohm's law is then

$$V = IZ.$$  

**Part C**

What is the total impedance of the circuit in Parts A and B?

**Enter your answer numerically in ohms.**

**ANSWER:**

$$Z = 0.532 \ \Omega$$

In ac circuits, $I$ and $V$ are not measured directly. Instead, ac ammeters and voltmeters are designed to measure the root-mean-square values of $I$ and $V$:

$$I_{\text{rms}} = \frac{I}{\sqrt{2}} \quad \text{and} \quad V_{\text{rms}} = \frac{V}{\sqrt{2}},$$

which can be used in the equation $V = IZ$ to yield

$$V_{\text{rms}} = I_{\text{rms}}Z.$$
Part D

If this circuit were connected to a standard 120 V ac outlet, what would the rms current in the circuit be?

Enter your answer numerically in amperes.

ANSWER:

$I_{\text{rms}} = 226 \ A$

The current is high because the total impedance is relatively low. Actually, plugging such a circuit into a 120-V outlet would most likely burn out the circuit elements.

A frequent application of $L-R-C$ ac circuits is the tuning mechanism in a radio. The $L-R-C$ ac circuit will have a resonant frequency that depends on both the inductance and capacitance of the circuit according to the formula

$$f_0 = \frac{1}{2\pi \sqrt{LC}}.$$  

This is the frequency at which the impedance is the smallest, which causes the largest current to appear in the circuit for a given $V_{\text{rms}}$. The radio picks up this resonant frequency and suppresses signals at other frequencies. A variable capacitor in this circuit causes the resonant frequency of the circuit to change. When you tune the radio you are adjusting the value of the capacitance in the circuit and hence the resonant frequency.

Part E

To see whether the $L-R-C$ ac circuit from Part A would be suitable for a tuner in a radio, find the resonant frequency of this circuit.

Enter your answer numerically in hertz.

ANSWER:

$f_0 = 31.5 \ Hz$

This frequency does not correspond to either the standard AM or FM band.

Phasors: Analyzing a Parallel AC Circuit

Description: This problem contains several questions, both qualitative and quantitative, directed at analyzing a parallel AC circuit using phasors. The circuit contains only a resistor and capacitor in parallel. It is recommended that the students solve the problem Phasors Explained before trying this one.

Learning Goal:

To understand the use of phasors in analyzing a parallel AC circuit.

Phasor diagrams, or simply phasors, provide a convenient graphical way of representing the quantities that change with time along with $\cos(\omega t + \phi)$. This makes them useful for analyzing AC circuits with their inherent phase shifts between voltage and current. If a quantity $I(t)$ changes with time as $I(t) = I_0 \cos(\omega t)$, a phasor is a vector whose length represents the amplitude $I_0$ (see the diagram). This vector is assumed to rotate counterclockwise with angular speed $\omega$; that way, the horizontal component of the vector represents the actual value $I(t)$ at any given moment.

In this problem, you will use the phasor approach to analyze an AC circuit. In answering the questions of this problem, keep the following in mind:

- For a resistor, the current and the voltage are always in phase.
- For an inductor, the current lags the voltage by $\frac{\pi}{2}$.
- For a capacitor, the current leads the voltage by $\frac{\pi}{2}$.
Part A

Phasors are helpful in determining the values of current and voltage in complex AC circuits. Consider this phasor diagram: The diagram describes a circuit that contains two elements connected in parallel to an AC source. The vector labeled $V_0$ corresponds to the voltage across both elements of the circuit. Based on the diagram, what elements can the circuit contain?

![Phasor Diagram]

ANSWER:

- two resistors
- two inductors
- two capacitors
- a capacitor and a resistor
- an inductor and a resistor
- a capacitor and an inductor

Part B

The phasor diagram from Part A describes a circuit that looks like the one in the figure: What are the respective amplitudes of the currents in the capacitor and the resistor in the diagram for Part A?

![Circuit Diagram]

ANSWER:

- $I_1 : I_2$
- $I_2 : I_1$

Part C

Find the amplitude of the current $I_0$ through the voltage source.

Express your answer in terms of the magnitudes of the individual currents $I_1$ and $I_2$.

**Hint 1. The parallel connection**

In a parallel circuit, the current through the voltage source is the sum of the individual currents. That rule, learned first for DC circuits, holds true for AC circuits, too: The law of conservation of charge is still valid! Therefore, at any instant, the current through the voltage source is the
Hint 2. The current through the voltage source as a vector sum

The current through the voltage source may also be represented by the component of a rotating vector whose length represents its amplitude $I_0$:

You should find the value of $I_0$ as the magnitude of the vector sum of the individual currents. Note that the phase angle between $I_1$ and $I_2$ is $\frac{\pi}{2}$.

\[ I_0 = \sqrt{I_1^2 + I_2^2} \]

**Part D**

What is the tangent of the phase angle $\phi$ between the voltage and the current through the voltage source?

Express $\tan(\phi)$ in terms of $I_1$ and $I_2$.

**Hint 1. The current through the voltage source as a vector sum**

The current through the voltage source may also be represented by the component of a rotating vector whose length represents its amplitude $I_0$:

You should find the value of $I_0$ as the magnitude of the vector sum of the individual currents. Note that the phase angle between $I_1$ and $I_2$ is $\frac{\pi}{2}$.

\[ \tan(\phi) = \frac{I_0}{I_1} \]

As you can see, phasors can be helpful in visualizing the processes in AC circuits.
Exercise 31.17

Description: You have a resistor of resistance 200 Ω and a 6.00-μF capacitor. Suppose you take the resistor and capacitor and make a series circuit with a voltage source that has a voltage amplitude of 30.0 V and an angular frequency of 250 (rad)/s. (a)... You have a resistor of resistance 200 Ω and a 6.00-μF capacitor. Suppose you take the resistor and capacitor and make a series circuit with a voltage source that has a voltage amplitude of 30.0 V and an angular frequency of 250 rad/s.

Part A
What is the impedance of the circuit?

ANSWER:

\[ Z = 696 \, \Omega \]

Part B
What is the current amplitude?

ANSWER:

\[ I = 4.31 \times 10^{-2} \, A \]

Part C
What is the voltage amplitude across the resistor?

ANSWER:

\[ V_R = 8.62 \, V \]

Part D
What is the voltage amplitudes across the capacitor?

ANSWER:

\[ V_C = 28.7 \, V \]

Part E
What is the phase angle \( \phi \) of the source voltage with respect to the current?

ANSWER:

\[ \phi = -73.3 \, ^\circ \]

Part F
Does the source voltage lag or lead the current?

ANSWER:

- [ ] the voltage lags the current
- [ ] the voltage leads the current

Part G
Construct the phasor diagram.

Draw the force vectors with their tails at the dot. The orientation of your vectors will be graded. The exact length of your vectors will not be graded but the relative length of one to the other will be graded.

ANSWER:

Alternating Voltages and Currents

**Description:** Find the rms voltage from the graph of the instantaneous voltage; then find the rms current and average power from a given value of R.

The voltage supplied by a wall socket varies with time, reversing its polarity with a constant frequency, as shown in the graph.

![Graph of voltage vs. time](image)

**Part A**

What is the rms value $V_{\text{rms}}$ of the voltage plotted in the graph?

Express your answer in volts.

**Hint 1.** RMS value of a quantity with sinusoidal time dependence

A quantity that varies with time as $x = x_{\text{max}} \sin \omega t$ (or $x = x_{\text{max}} \cos \omega t$) has a maximum value equal to $x_{\text{max}}$ and an rms value given by

$$x_{\text{rms}} = \frac{x_{\text{max}}}{\sqrt{2}}.$$

**Hint 2.** Find the maximum value of the voltage
What is the maximum value $V_{\text{max}}$ of the voltage plotted in the graph?

Express your answer in volts.

ANSWER:

$V_{\text{max}} = 170 \text{ V}$

This is the standard rms voltage supplied to a typical household in North America.

**Part B**

When a lamp is connected to a wall plug, the resulting circuit can be represented by a simplified AC circuit, as shown in the figure. Here the lamp has been replaced by a resistor with an equivalent resistance $R = 120 \Omega$. What is the rms value $I_{\text{rms}}$ of the current flowing through the circuit?

Express your answer in amperes.

**Hint 1. Ohm's law in AC circuits**

Ohm's law can still be applied to an AC circuit, provided the values used to describe all physical quantities are consistent. For example, Ohm's law can be written using maximum values of voltage and current, or alternatively using rms quantities.

ANSWER:

$I_{\text{rms}} = \frac{170}{\sqrt{3}} \frac{\text{V}}{R} = 1.00 \text{ A}$

**Part C**

What is the average power $P_{\text{avg}}$ dissipated in the resistor?

Express your answer in watts.

**Hint 1. Average power in an AC circuit**

The average power $P_{\text{avg}}$ dissipated in a resistor with resistance $R$ is given by

$$P_{\text{avg}} = \overline{I^2_{\text{rms}}} R,$$

where $I_{\text{rms}}$ is the rms value of the current flowing through the resistor. Note that the average power can also be expressed as

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R},$$

where $V_{\text{rms}}$ is the rms voltage supplied to the resistor.
The instantaneous power dissipated in the resistor can be substantially higher than the average power. However, since the voltage supplied to the resistor varies in time, so does the instantaneous power. Therefore, a better estimate of the energy dissipated in an AC circuit is given by the average power. For example, the power rating on light bulbs is in fact the average power dissipated in the bulb.

Exercise 31.23
Description: An L-R-C series circuit \( L = \text{## H} \), \( R = \text{## \Omega} \), and \( C = \text{## \mu F} \) carries an rms current of \( I \) with a frequency of \( f \). (a) What is the phase angle? (b) What is the power factor for this circuit? (c) What is the impedance of the circuit? (d) What is the rms...

An L-R-C series circuit \( L = 0.115 \text{ H} \), \( R = 237 \text{ \Omega} \), and \( C = 7.32 \text{ \mu F} \) carries an rms current of 0.445 A with a frequency of 405 Hz.

Part A
What is the phase angle?

ANSWER:

\[
\phi = \arccos \left( \frac{R}{\sqrt{R^2 + \left( \frac{2\pi f L - \frac{1}{2\pi f C}}{2} \right)^2}} \right) = 0.790 \text{ radians}
\]

Part B
What is the power factor for this circuit?

ANSWER:

\[
\frac{R}{\sqrt{R^2 + \left( \frac{2\pi f L - \frac{1}{2\pi f C}}{2} \right)^2}} = 0.704
\]

Part C
What is the impedance of the circuit?

ANSWER:

\[
Z = \sqrt{R^2 + \left( \frac{2\pi f L - \frac{1}{2\pi f C}}{2} \right)^2} = 337 \text{ \Omega}
\]

Part D
What is the rms voltage of the source?

ANSWER:

\[
V_{\text{rms}} = I \sqrt{R^2 + \left( \frac{2\pi f L - \frac{1}{2\pi f C}}{2} \right)^2} = 150 \text{ V}
\]
Part E
What average power is delivered by the source?
ANSWER:
\[
P = \frac{I^2 \sqrt{R^2 + \left(\frac{2\pi f L - \frac{1}{2\pi f C}}{2\pi f C}\right)^2} R}{\sqrt{R^2 + \left(\frac{2\pi f L - \frac{1}{2\pi f C}}{2\pi f C}\right)^2}} = 46.9 \text{ W}
\]

Part F
What is the average rate at which electrical energy is converted to thermal energy in the resistor?
ANSWER:
\[
P_R = \frac{I^2 \sqrt{R^2 + \left(\frac{2\pi f L - \frac{1}{2\pi f C}}{2\pi f C}\right)^2} R}{\sqrt{R^2 + \left(\frac{2\pi f L - \frac{1}{2\pi f C}}{2\pi f C}\right)^2}} = 46.9 \text{ W}
\]

Part G
What is the average rate at which electrical energy is dissipated (converted to other forms) in the capacitor?
ANSWER:
0 W

Part H
What is the average rate at which electrical energy is dissipated (converted to other forms) in the inductor?
ANSWER:
0 W

Exercise 31.33
Description: In an L-R-C series circuit, \(L=0.280 \text{ H}\) and \(C=4.00 \mu \text{ F}\). The voltage amplitude of the source is 120 V. (a) What is the resonance angular frequency of the circuit? (b) When the source operates at the resonance angular frequency, the current amplitude...

In an L-R-C series circuit, \(L = 0.280 \text{ H}\) and \(C = 4.00 \mu \text{ F}\). The voltage amplitude of the source is 120 V.

Part A
What is the resonance angular frequency of the circuit?
ANSWER:
\[\omega_0 = 945 \text{ rad/s}\]

Part B
When the source operates at the resonance angular frequency, the current amplitude in the circuit is 1.70 A. What is the resistance \(R\) of the resistor?
ANSWER:
Part C
At the resonance angular frequency, what are the peak voltages across the inductor?

ANSWER:

\[ V = 450 \ \text{V} \]

Part D
At the resonance angular frequency, what are the peak voltages across the capacitor?

ANSWER:

\[ V = 450 \ \text{V} \]

Part E
At the resonance angular frequency, what are the peak voltages across the resistor?

ANSWER:

\[ V = 120 \ \text{V} \]

The Resonance Peak

Description: Short quantitative problem on resonance in a series RLC circuit. Based on Young/Geller Quantitative Analysis 22.5.

In this problem we consider the resonance curve for a circuit with electrical components \( L \), \( R \), and \( C \) and resonant frequency \( \omega_0 \).

Part A
For given values of \( R \) and \( C \), if you double the value of \( L \), how does the new resonance curve differ from the original one?

**Hint 1. How to approach the problem**

In a series R-L-C circuit, the resonance frequency is determined by \( L \) and \( C \), whereas the peak value of the current in the circuit depends only on \( R \). Since the value of \( R \) is being kept constant, the peak of the resonance curve is unchanged.

To solve this problem, use proportional reasoning to find a relation between the resonance frequency \( f_0 \) and inductance \( L \).

- Find the simplest equation that contains these variables and other known quantities from the problem.
- Write this equation twice, once to describe the original circuit and again for the circuit with greater inductance.
- You then need to write each equation with all the constants on one side and the variables on the other. Since the variable is \( f_0 \) in this problem, you will write the equations in the form \( f_0 = \cdots \).
- To finish the problem you need to compare the two cases presented. For this question you should find the ratio of the resonance frequency of the original circuit to that of the circuit with greater inductance.

**Hint 2. Resonance frequency**

In a series R-L-C circuit, the resonance frequency \( f_0 \) is given by

\[ f_0 = \frac{1}{2\pi\sqrt{LC}}. \]

**Hint 3. Find an expression for the new resonance frequency**

Write an expression for the resonance frequency \( f_1 \) of the circuit when the inductance \( L \) is doubled?

Express your answer in terms of some or all of the variables \( R \), \( L \), and \( C \).
In an R-L-C circuit, the resonance peak depends only on $R$, while the resonant frequency is determined by both $L$ and $C$. In particular, for a given value of $C$, the resonance frequency is inversely proportional to the square root of $L$. Similarly, for a given value of $L$, the resonance frequency is inversely proportional to the square root of $C$.

**Part B**

For given values of $R$ and $C$, if you double the value of $L$, how does the new rms current at resonance $I_{\text{rms}}$ differ from its original value? Assume that the voltage amplitude of the ac source is the same.

**Hint 1. How to approach the problem**

Recall that in an ac circuit, for a given voltage, the rms current is always inversely proportional to the impedance of the circuit. This holds also at resonance.

To solve this problem, use proportional reasoning and find a relation between the rms current $I_{\text{rms}}$ and the impedance $Z$.

- Find the simplest equation that contains these variables and other known quantities from the problem.
- Write this equation twice, once to describe the original circuit and again for the circuit with greater inductance.
- You need to write each equation so that all the constants are on one side and the variables are on the other. Since the variable is $I_{\text{rms}}$ in this problem, you will write the equations in the form $I_{\text{rms}} = \cdots$.
- To finish the problem you need compare the two cases presented. For this question you should find the ratio of the rms current in the original circuit to that in the circuit with greater inductance.

**Hint 2. $V_{\text{rms}}$ across an ac circuit**

In a series R-L-C circuit, the rms voltage $V_{\text{rms}}$ across the circuit is always proportional to the rms current $I_{\text{rms}}$ in it. If $Z$ is the impedance of the circuit, then

$$V_{\text{rms}} = I_{\text{rms}}Z.$$

**Hint 3. Find the impedance of the circuit**

What is the impedance $Z$ of a series R-L-C circuit at resonance?

Express your answer in terms of some or all of the variables $R$, $L$, $C$, and $\omega_0$.

**Hint 1. Impedance of a series R-L-C circuit**

The impedance $Z$ of a series R-L-C circuit is a function of $R$, $L$, $C$, and the angular frequency $\omega$. It can be expressed as

$$Z = \sqrt{R^2 + (X_L - X_C)^2},$$

where $X_L$ and $X_C$ are the respective inductive and capacitive reactances of the circuit.

**Hint 2. Condition for resonance**

At resonance, the inductive and capacitive reactances must be equal. That is, $X_L = X_C$. 

ANSWER:

$$f_1 = \frac{1}{2\pi\sqrt{LC}}$$
At resonance, the impedance of the circuit does not depend on $L$. Thus, for a given voltage, will $I_{\text{rms}}$ change when $L$ is increased?

ANSWER:

- $I_{\text{rms}}$ is twice as great.
- $I_{\text{rms}}$ is half as great.
- $I_{\text{rms}}$ is $1/\sqrt{2}$ times as great.
- $I_{\text{rms}}$ is unchanged.

In a series R-L-C circuit, for a given voltage, the rms current is always inversely proportional to the circuit impedance. Since at resonance the impedance depends only on $R$, the rms current in the circuit remains constant when either $L$ or $C$ is changed. Note that this is true only at resonance.

Secondary Voltage and Current in a Transformer Ranking Task

**Description:** Rank the current and voltage in the secondary coil of different transformers. (ranking task)

Six transformers have the rms primary voltages ($V_p$), number of primary turns ($N_p$), and number of secondary turns ($N_s$) listed below.

**Part A**

Which of the transformers are step-up transformers? Which of the transformers are step-down transformers?

Place the appropriate transformers into the two categories listed below.

**Hint 1. Step-up and step-down transformers**

A transformer is referred to as step-up if the secondary voltage is larger than the primary voltage. Similarly, a transformer is referred to as step-down if the secondary voltage is smaller than the primary voltage.

**Hint 2. Turns ratio**

The turns ratio, $N_s/N_p$, is the factor that determines the characteristics of a transformer. If the ratio is greater than one, the transformer “steps up” the voltage by this factor. If it is less than one, it “steps down” the voltage.

ANSWER:
Part B

Rank the transformers on the basis of their rms secondary voltage.

Rank from largest to smallest. To rank items as equivalent, overlap them.

**Hint 1. Voltage and number of turns**

The proportionality constant between voltage and number of turns is the same on both sides of the transformer.

**Hint 2. Secondary voltage**

Since the proportionality constant between voltage and number of turns is the same on both sides of the transformer, we can set the ratios \( V/N \) on both sides equal to each other:

\[
\frac{V_p}{N_p} = \frac{V_s}{N_s},
\]

or

\[
V_s = \left( \frac{N_s}{N_p} \right) V_p.
\]

Therefore, the secondary voltage is just the product of the primary voltage and the turns ratio.

ANSWER:
Part C

100 A of rms current is incident on the primary side of each transformer. Rank the transformers on the basis of their rms secondary current.

Rank from largest to smallest. To rank items as equivalent, overlap them.

**Hint 1. Energy conservation**

Since energy flow (power) in a circuit is given by

\[ P = IV, \]

energy conservation requires that any increase in voltage be accompanied by a corresponding decrease in current, and vice versa. Thus, a step-up transformer will step down current.

**Hint 2. Stepping down current in a step-up transformer**

Since voltage is stepped up by the turns ratio,

\[ V_s = \left( \frac{N_s}{N_p} \right) V_p, \]

conservation of energy ensures that current is stepped down by the same ratio. This can be written as

\[ I_s = \left( \frac{N_p}{N_s} \right) I_p. \]

ANSWER:
Exercise 31.9

**Description:** (a) What is the reactance of an inductor with an inductance of L at a frequency of f? (b) What is the inductance of an inductor whose reactance is $X_L$ at a frequency of f? (c) What is the reactance of a capacitor with a capacitance of C at a...

**Part A**

What is the reactance of an inductor with an inductance of 3.00 H at a frequency of 80.0 Hz?

**ANSWER:**

$$X_L = \frac{2\pi f L}{2\pi f} = 1510 \Omega$$

**Part B**

What is the inductance of an inductor whose reactance is 12.8 Ω at a frequency of 80.0 Hz?

**ANSWER:**

$$L = \frac{X_L}{2\pi f} = 2.55 \times 10^{-2} \text{ H}$$

**Part C**

What is the reactance of a capacitor with a capacitance of $4.42 \times 10^{-6}$ F at a frequency of 80.0 Hz?

**ANSWER:**

$$X_C = \frac{1}{2\pi f C} = 450 \Omega$$

**Part D**
What is the capacitance of a capacitor whose reactance is 129 Ω at a frequency of 80.0 Hz?

\[ C = \frac{1}{2\pi f X_C} = 1.54 \times 10^{-5} \text{ F} \]

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**Exercise 31.34**

**Description:** You plan to take your hair blower to Europe, where the electrical outlets put out V1 instead of the V2 seen in the United States. The blower puts out P at V2. (a) What could you do to operate your blower via the V1 line in Europe? (b) What current...

You plan to take your hair blower to Europe, where the electrical outlets put out 240 V instead of the 120 V seen in the United States. The blower puts out 1600 W at 120 V.

**Part A**

What could you do to operate your blower via the 240 V line in Europe?

**ANSWER:**

- Use a step_up transformer with \( N_2 / N_1 = 2 \).
- Use a step_down transformer with \( N_2 / N_1 = 1/2 \).

**Part B**

What current will be drawn from the 240 V line when the dryer is connected to the 120 V side of the transformer?

Express your answer using two significant figures.

**ANSWER:**

\[ I = I = 6.7 \text{ A} \]

**Part C**

What resistance connected directly to the 240 V line would draw the same power as the dryer in part B?

Express your answer using two significant figures.

**ANSWER:**

\[ R_{eff} = R_{eff} = 36 \text{ } \Omega \]