The evolution of crack-tip stresses during a fatigue overload event

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Abstract

The mechanisms responsible for the transient retardation or acceleration of fatigue crack growth subsequent to overloading are a matter of intense debate. Plasticity-induced closure and residual stresses have often been invoked to explain these phenomena, but closure mechanisms are disputed, especially under conditions approximating to generalised plane strain. In this paper we exploit synchrotron radiation to report very high spatial resolution two-dimensional elastic strain and stress maps at maximum and minimum loading measured under plane strain during a normal fatigue cycle, as well as during and after a 100% overload event, in ultra-fine grained AA5091 aluminium alloy. These observations provide direct evidence of the material stress state in the vicinity of the crack-tip in thick samples. Significant compressive residual stresses were found both in front of and behind the crack-tip immediately following the overload event. The effective stress intensity at the crack-tip was determined directly from the local stress field measured deep within the bulk (plane strain) by comparison with linear elastic fracture mechanical theory. This agrees well with that nominally applied at maximum load and 100% overload. After overload, however, the stress fields were not well described by classical $K$ fields due to closure-related residual stresses. Little evidence of overload closure was observed sometime after the overload event, in our case possibly because the overload plastic zone was very small.

Keywords: Plasticity-induced closure; Stress intensity factor; crack-tip stress field; Overload; Retardation

1. Introduction

While most fatigue data is collected in a constant nominal crack-tip stress intensity range ($\Delta K = K_{\text{max}} - K_{\text{min}}$), in practice it is commonplace for engineering components to experience a spectrum of loads. With this in mind, considerable research has been undertaken to characterise the transient crack growth behaviour following individual over- or underload spikes in loading. The current state of knowledge can be summarised thus [1].

(i) Overloads retard, while underloads accelerate, the rate of crack growth rate relative to the underlying constant amplitude rate.
(ii) A number of cycles ($N_d$) must be applied before the original steady-state crack growth rate is re-established following an over- or underload event.
(iii) The extent of the retardation effect depends on the overload ratio (OLR, $K_{\text{OMax}}/K_{\text{Max}}$), the baseline value of $\Delta K$ and the load ratio ($R$).
(iv) Overloads can produce a very short initial acceleration phase prior to prolonged retardation.

These effects have previously been explained in terms of plasticity-induced closure, crack-tip blunting, residual...
compressive stresses and crack-tip branching or deflection. Plasticity-induced closure explanations have existed almost as long as crack closure has been proposed as a crack growth retardation mechanism [2]. The argument is based on the fact that the overload creates a larger stretch in the wake of the fatigue crack, which causes crack closure as the crack-tip progresses through the overload plastic zone. Since the crack must first progress into the overload plastic zone, this would explain why retardation is delayed.

Because of the larger plastic zone and the capacity for inward movement of material at the crack-tip under plane stress, one might expect plasticity-induced closure to be more significant in plane stress than generalised plane strain [3]. Under plane stress a potential mechanism of material transfer is obvious. Since out-of-plane deformation is not constrained, material can be transferred from the thickness direction to the loading direction. However, the mechanism of material transfer postulated for plane stress is not admissible for plane strain. By definition, no net out-of-plane contraction can occur and, therefore, it has been suggested that there can be no net axial stretch of material in the plastic wake behind the crack-tip, which implies no plasticity-induced crack closure [4]. The existence of plasticity-induced crack closure under plane strain conditions has thus been a topic of intense debate [5]. Fleck and Newman have shown that closure does not occur for a bend specimen under plane strain conditions, while it does occur for the middle crack tension geometry under plane strain [6]. This may be due to the fact that the latter has a compressive $T$ stress, while the former has a tensile $T$ stress. Contrary to earlier work [7], Sadananda et al. [1] dismissed plasticity-induced crack closure, arguing that plasticity always acts to open rather than close the crack.

The main problem with the crack-tip blunting explanation is that while it can reduce the stress intensity at the crack-tip, it cannot easily explain delayed retardation. It has been argued that reverse yielding ahead of the crack-tip increases the size and magnitude of the compressive residual stress zone, thereby retarding crack growth. However, the largest residual stresses will occur in the immediate vicinity of the crack-tip, so it is difficult to explain delayed retardation [8]. Further, the extent of the retardation phase seems to be larger than the likely extent of the compressive zone [9]. Especially for alloys with a propensity for planar slip, tensile overloads can promote crack deflection away from mode I (pure crack-opening). As a result, there is a short burst of accelerated growth, followed by retardation due to the lower $\Delta K_{eff}$. However, such mechanisms are not generally observed at the crack-tip, while overload retardation is widely observed.

Regarding residual stress arguments, simple models such as Dugdale’s strip model [10] have been developed to explain the evolution of crack-opening stress. These assume that crack-tip plasticity occurs in thin strips, with crack-tip plastic displacements and crack advance calculated step-wise by considering the region broken down into strips to simulate formation of the plastic wake. The triaxial constraints at the crack-tip are introduced into such models using a constraint factor (for which ideal values are 1 or 3 for plane stress and plane strain, respectively).
The approach of Beretta and Carboni [11] is shown schematically in Fig. 1 where \( y_x, z_t \), and \( z_w \) are the constraint factors for compressive and tensile yielding and for compressive yielding of the wake, respectively.

To date, experimental measurements of crack closure have relied mostly on either: (i) measuring some secondary property of the cracked body, such as compliance or electrical resistance, or (ii) measurement of crack opening displacements on the surface of the cracked body. There are significant difficulties in interpreting secondary data in terms of crack-tip stresses, as the surface of a cracked body experiences conditions of plane stress, in contrast to the bulk which experiences conditions more akin to plane strain.

In summary, the mechanisms of overload fatigue crack retardation are still the subject of debate. Some of the mechanisms relate to events in the crack wake, some ahead of the crack-tip. Crack-tip residual stresses under plane stress have been measured at the surface using X-rays [12]. While the measurement of stress fields in the bulk of cracked samples by neutron diffraction has been possible for some years [13–15], the spatial resolution (~1 mm) has been insufficient to resolve the immediate crack-tip field. Recently, very narrow, but intense, beams of hard X-rays have become available at third generation synchrotron X-ray sources. This has opened up the way for much higher spatial resolutions, enabling crack-tip strains to be evaluated [16–19]. Recently, elegant synchrotron X-ray diffraction (SXRD) experiments have been directed towards overload phenomena on 4 mm thick samples [20], showing evidence of the overload on crack-tip stresses for cracks showing subsequent retardation. SXRD also enables the study of cracks in much thicker samples [17,21], presenting a unique opportunity to non-destructively determine the crack-tip elastic strain field under plane strain, i.e. in both the plastic and \( K \)-dominant zones. As high energy X-ray diffraction utilizes small diffraction angles the very high (lateral) spatial resolution can usually only be achieved in two orthogonal directions within bulk specimens. However, if sufficiently thick cracked specimens are used then, in principle, the full crack-tip stress field can be determined, since the conditions at the centre of such specimens approximate well to generalised plane strain.

This paper reports what we believe to be the first measurements of the two-dimensional crack-tip stress field in the plane strain region of thick samples at a resolution of tens of microns. In order to examine the extent to which residual stresses arise during conventional fatigue and subsequent to an overload results are reported at various points within the cycle for baseline fatigue, as well as during, after and well beyond a 100% overload event.

2. Experimental details

2.1. The material studied

The material used in this study is a powder metallurgy Al–Li alloy based on alloy AA5091 obtained from Aerospace Metal Composites Ltd., UK. The alloy composition was Al, 4.0% Mg, 1.2% Li, 1.0% C, 0.5% O and was produced from powders using a mechanical alloying process followed by hot isostatic pressing [22,23]. The billet was supplied as an upset-forged 42 mm thick plate in the as forged (T1) condition. The material exhibited a yield stress of 450 MPa and a tensile strength of 505 MPa, values similar to those previously reported for AA5091 in the T1 condition [24,25]. The tensile performance of Al–Mg–Li–C alloys is derived from significant Hall–Petch strengthening due to their very fine grain structure, as well as solution and dispersion hardening. They typically possess a high Mg concentration in comparison with heat treatable Al–Li alloys, but the Li concentration is deliberately kept below 1.5%, to prevent formation of the age hardening \( \delta' \) phase (Al\(_3\)Li). During mechanical alloying surface layers of oxides on the powder particles are repeatedly grown, then fractured and dispersed within the particles. The grains, which are typically smaller than 1 \( \mu \)m, contain significant distributions of fine (50–100 nm) precipitates, which are believed to be Al\(_4\)C\(_3\), Al\(_2\)O\(_3\) and MgO. The Al\(_2\)O\(_3\) and MgO phases derive from fragmentation of the surface oxides from the as received materials. The Al\(_4\)C\(_3\) phase, in contrast, derives only from reaction between the aluminium and the process control agent used to aid the sintering process [22,26]. The ultrafine grains promote homogeneous deformation and prevent crystallographic cracking. This fine microstructure also endows the alloy with low levels of crack closure and yield stress-insensitive fatigue crack propagation behaviour so that the properties of the as forged T1 materials are indistinguishable from those of the fully heat-treated condition [27]. The very small grain size makes it an ideal material for a high spatial resolution investigation by energy dispersive X-ray diffraction, since it allows very narrow slits to be used while still providing a good powder diffraction pattern.

2.2. The specimens

Three 12 mm thick compact tension test pieces were used, as defined in the fatigue crack propagation standard ASTM E647. Fatigue cracks were initiated in all the specimens and grown under conditions of constant nominal stress intensity range (\( AK_I = 6 \text{MPa}\sqrt{\text{m}}, R = 0.1 \)), with \( K_{I_{\text{Max}}} = 6.6 \text{MPa}\sqrt{\text{m}} \) and \( K_{I_{\text{Min}}} = 0.6 \text{MPa}\sqrt{\text{m}} \) using an MTS servo-hydraulic testing system. The crack length was monitored with a traveling microscope and the compliance of the specimen was recorded using a back face strain gauge. The three specimens were examined under the following conditions (see also Table 1):

- For specimen S1 (F) the crack was grown to a length of 26 mm.
- For specimen S2 (FO) the crack was grown to 27.7 mm and then a 100% overload corresponding to 13.2 MPa/\( \sqrt{\text{m}} \) (2\( K_{I_{\text{Max}}} \)) was applied.
For specimen S3 (FOF) the crack was grown to 26.6 mm, where a 100% overload (13.2 MPa) was applied, after which the crack was extended by a further 0.1 mm, resulting in a final crack length of 26.7 mm.

For crack-tip stress field mapping these were transferred to the synchrotron X-ray beam line and loaded in situ using a modified Hounsfield tensometer mounted vertically (Fig. 2a). For each test specimen the load was applied to the correct level using the back face strain and the strain gauge calibration obtained previously in the laboratory.

Partly as a result of the very small grain size, crack morphologies under mode I loading in this material are typically very straight with the low surface roughness giving rise to very low levels of roughness-induced closure. It should be noted that at $K_{\text{I max}}$ and $2K_{\text{I max}}$ the static forward tensile plastic zones under plane strain would be expected to be approximated by $r_y \approx K_{\text{I max}}^2 / \bar{\sigma}_y^2$ [28], such that for this alloy $r_y$ is equal to 11 µm and 45 µm, for $K_{\text{I max}}$ and $2K_{\text{I max}}$, respectively, while the reverse compressive plastic zones would be approximately four times smaller.

2.3. The diffraction geometry and experimental set-up

The experimental set-up comprised two solid-state X-ray detectors mounted at $2\theta = 5^\circ$, one horizontally and one vertically offset. This configuration allowed the measurement of two essentially perpendicular directions of strain simultaneously (Fig. 2a; cf. [19]). The beam apertures were fixed at 25 µm in the vertical and horizontal directions to define the lateral spatial resolutions some 8 × better than the best that had been achieved previously [18]. For each detector the receiving slits near the detector and those near the sample were both set at 40 µm. At low scattering angles the strain resolution was sub-optimal, however, the use of an ultrafine grained material and the large Ewald sphere at high photon energies, as well as the multiple peak refinement, helped to maintain good diffraction conditions and good strain resolution. The detectors and receiving slits were aligned and focused using the concentric tip of a small 0.5 mm diameter steel pin. The alignment between the detectors was precise to within 20 µm.

The strains were measured on a map of 25 µm pitch within 1 mm$^2$ of the crack-tip, and on a coarser 100 µm pitch over $2 \times 2$ mm$^2$ further from the crack-tip. The approximate location of the crack-tip had previously been determined optically on the surface of the test piece and set initially to (0, 0). The measurement positions are indicated by crosses overlaid on the strain map in Fig. 3a. (See below for a detailed explanation of this figure.) The reference frame was maintained during in situ loading by a 0.5 mm diameter steel pin that was inserted at a specified location near the notch of the computer tomography (CT) specimen and scanned before each map.

A typical spectrum and its best-fit refinement (using GSAS, see below) in energy dispersive mode are shown in Fig. 2b. The spectra were very clear with low backgrounds and no indication of strong texture. Initially a Gaussian-type peak profile was used for the refinement, but, as indicated by the difference curve in Fig. 2b, there was a Lorentzian contribution to the profile. In order to access the more complex peak profiles available in GSAS, the experimental data were first converted to a pseudo-angular dispersive scale [29]. Fitting was then undertaken.
using GSAS [30] in intensity extraction mode [31], which fits all peak positions to the required parameters, in our case simply the lattice parameter. Typically only the region 50–150 keV (or rather its equivalent in angular dispersive terms) was included in the refinements. The strain was then calculated using the relative variation of the refined lattice parameter \(a\) across the sample in the usual way, i.e.
\[
\varepsilon = \frac{a - a_0}{a_0}.
\]
This approach has the advantage that, by including many peaks in the refinement, the measured strains are likely to be representative of the macroscopic response, free from significant elastic or plastic anisotropy effects [32,33]. The unstrained lattice parameter \(a_0\) was obtained from the averaged far field measurement in the unloaded state [34]. This value agreed well with that obtained by applying a stress balancing condition on a suitable part of the data, e.g. in the crack opening direction along the crack plane to the back face. For any subsequent diffraction peak width analysis a dedicated single peak fitting routine was employed.

The counting times were of the order of 120 s per point, being limited primarily by the low counting rate (40 kHz) of the detectors rather than the diffracted flux. The strain uncertainty (the uncertainty in the lattice parameter refinement) at each point was of the order of 10\(^{-5}\), being an order of magnitude smaller than the point-to-point variation, which is due to material (microstructural) and instrumental factors. Stress in any three orthogonal directions can be calculated from the corresponding elastic strains using the relationship:

\[
\sigma_{ij} = \frac{E}{1 + \nu} \left[ \varepsilon_{ij} + \frac{\nu}{1 - 2\nu} (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) \right] \tag{1}
\]

Following the arguments summarised in Suresh [35], a crack in a fracture mechanics specimen is likely to undergo plane strain conditions if its thickness \(B\) is greater than \(2.5 (K/\sigma_y)^2\), which gives for test specimen S1 \(B > 0.53\) mm and for specimen S2 containing the overload crack \(B > 2.15\) mm. These thicknesses will be even smaller for fatigue and, thus, it is justified to calculate the stress field in both specimens using a plane strain approximation. Here we take this to mean that the elastic strain is in a state of plane strain \([\varepsilon_{zz} = 0, \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})]\).

2.4. Determination of crack-tip stress intensity

The advent of surface strain/displacement mapping techniques [36,37] opened the way for the development of algorithms for the determination of crack-tip stress inten-

![Image](https://via.placeholder.com/150)

Fig. 3. The raw measured data (top) and best fit (bottom) elastic strain fields \((10^{-6})\) for (a) and (c) the crack growth \((\varepsilon_{xx})\) and (b) and (d) the crack opening \((\varepsilon_{yy})\) directions at \(K_{I,\text{Max}} = 6.6\) MPa/\(\text{m}\) for S1. In (a) the crosses mark the coarse and fine scan measurement locations. The LEFM best fit crack-tip strains correspond to a crack-tip stress intensity of \((K_{\text{IT,Max}} = 6.11\) MPa/\(\text{m}, K_{\text{IT,Min}} = 0.33\) MPa/\(\text{m})\).
inity factors directly from the (total) strain \([38–41]\) or from the displacement \([42]\) fields thus measured. The advantage of these methods is that one can determine the stress intensity experienced by the crack-tip as against that nominally applied; the disadvantage is that most displacement or strain mapping methods are applicable only at the surface of metals and local strains derived from displacements tend to be rather noisy due to numerical differentiation. By coupling the theoretical linear elastic fracture mechanics (LEFM) crack-tip strain field to the (essentially) elastic strain fields measured at mid thickness by SXRD we have been able to infer the effective crack-tip stress intensity under plane strain directly from the crack-tip stress field to compare against that nominally applied.

3. Experimental procedure and results

Strain maps over the mid plane were collected at various stages of loading. For the fatigued test specimen (S1) at \(K_{IMin} = 0.6 \text{ MPa}\sqrt{\text{m}}\), \(K_{IMean} = 3.6 \text{ MPa}\sqrt{\text{m}}\), \(K_{IMax} = 6.6 \text{ MPa}\sqrt{\text{m}}\), and \(K_{IOMax} = 2K_{IMax} = 13.2 \text{ MPa}\sqrt{\text{m}}\), and for the fatigued overloaded (FOF) specimen (S3) at \(K_{IMin} = 0.6 \text{ MPa}\sqrt{\text{m}}\), \(K_{IMean} = 3.6 \text{ MPa}\sqrt{\text{m}}\), \(K_{IMax} = 6.6 \text{ MPa}\sqrt{\text{m}}\).

3.1. Baseline fatigue (S1)

The elastic strain fields recorded for the baseline fatigue cracked specimen (S1) at \(K_{IMax}\) are shown for the crack opening \([\varepsilon_{yy}(x, y)]\) and crack growth \([\varepsilon_{xx}(x, y)]\) directions in Fig. 3a and b. The measured mid thickness strain fields have been compared against the predicted LEFM crack-tip strain distributions to derive the best fit mode I and mode II stress intensity factors. These are summarised in Table 1 and the best fit elastic fields are shown in Fig. 3 (bottom). From Table 1 it can be seen that the inferred crack-tip stress intensities are in excellent agreement with those nominally applied both at maximum load and for the overload case. These results suggest that, as one would expect, any prior residual stresses have a negligible effect on the crack-tip stress intensity at \(K_{IMax}\) and \(K_{IOMax}\) and that the measured fields shown in Fig. 3 are essentially what one would expect on the basis of LEFM.

The strain fields \([\varepsilon_{yy}(x, y), \varepsilon_{xx}(x, y)]\) measured at \(K_{IMax}\) for S1 have been converted to stress using Eq. (1) in combination with the plane-strain assumption and are

![Fig. 4](image_url)

Fig. 4. Measured mid thickness (a) crack growth \((\sigma_{xx})\), (b) crack opening \((\sigma_{yy})\), (c) through thickness \((\sigma_{zz})\) direction and (d) hydrostatic \((\sigma_H)\) stress fields for S1 loaded to \(K_{IMax}\).
shown in Fig. 4, akin to Fig. 3. It can be seen that the ‘butterfly’ lobes evident in the crack opening strain map in Fig. 3b are not present in the corresponding crack-opening stress map (Fig. 4b). Significant peak stresses were found for all three stress components immediately ahead of the crack, creating a triaxial tensile field as expected \[\sigma_{yy}(x, y) > \sigma_{zz}(x, y) > \sigma_{xx}(x, y)\] ahead of a crack-tip in plane strain.

Although the measured von Mises stress was only around 140 MPa, it should be noted that the plastic zone is expected to be somewhat smaller than the gauge volume, so that the peak stresses recorded at the crack-tip will be depressed by averaging over the gauge volume.

The elastic strain and stress fields recorded upon unloading from \(K_{I_{\text{Max}}}\) to the minimum load \(K_{I_{\text{Min}}} = 0.6 \text{ MPa}\) are plotted in Fig. 5 at somewhat higher magnification.

It is clear from Fig. 5a and b that, as one might expect, the elastic strains in the sample are much smaller at \(K_{I_{\text{Min}}}\). It is not possible to identify a distinct reverse plastic zone in the immediate vicinity of the crack-tip. Our results are in contrast to those recorded by Akiniwa et al. [43] using surface X-ray diffraction (plane stress) for fine grained steel, who observed a residual stress consistent with a marked reverse plastic zone in accordance with closure models [44]. There are two possible explanations; either the reverse plastic zone introduced here is too small to create sufficient residual stresses to be recorded by a 25 µm gauge dimension, noting that for the previous plane stress case the plastic zone would be larger, or because closure is inherently difficult under plane strain.

The calculated residual stresses at \(K_{I_{\text{Min}}}\) are shown in Fig. 5. There is some evidence of a 50–100 µm wide compressive stress across the wake of the crack (Fig. 5d). However, this should be treated with caution given the observation by Croft et al. [18] of similar low level ‘compressive’ strains on the crack surfaces, even after splitting the sample in half. In their case these could have resulted from intergranular strains arising from crack-tip plasticity associated with using the single diffraction peak (3 2 1). For the (3 2 1) plane the observed sense (compressive) of intergranular strain in the crack opening direction was in line with the axial plastic tension and the broadened diffraction peak profile in this region. Such an explanation is less likely here as we have used the ‘full pattern profile refinement’ to obtain strain and, hence, our measurements are less likely to be affected by type II intergranular strains. Alternatively, the ‘stresses’ could be due to a slight error in strain free lattice spacing, however, since the stress in the crack opening direction (Fig. 5c) shows no such feature it may be a real effect of low level contact stress. The observation of tensile crack opening and crack growth direction stresses after the crack position might confirm very low level closure.

Fig. 5. Close-up of the mid thickness elastic strain and stress fields at \(K_{I_{\text{Min}}} = 0.6 \text{ MPa}\) for S1. The elastic strains \((10^{-6})\) in the (a) crack-growth \((e_{xx})\) and (b) crack opening \((e_{yy})\) directions. Inferred stresses in (c) the crack growth \((\sigma_{xx})\) and (d) the crack opening \((\sigma_{yy})\) directions. It should be remembered that the strains were derived by multiple peak refinements.
3.2. Overload case (S2)

The crack-opening stress fields for the fatigued and then 100% overloaded crack (FO) at $2K_{I_{\text{Max}}}$ (13.2 MPa/\(\sqrt{m}\)), $K_{I_{\text{Mean}}}$ (3.6 MPa/\(\sqrt{m}\)) and $K_{I_{\text{Min}}}$ (0.6 MPa/\(\sqrt{m}\)) are shown in Fig. 6. The stress field at $2K_{I_{\text{Max}}}$ is clearly much more intense than that shown in Fig. 3 for $K_{I_{\text{Max}}}$. Indeed, the stress intensity representative of the crack-tip stress field at $2K_{I_{\text{Max}}}$ was determined directly from the crack-tip strain field and the inferred stress intensity factor was in excellent agreement with that nominally applied (see Table 1). Further, it is clear that the effective stress intensity was found with a good level of uncertainty. While the peak stress was considerably larger than both the typical yield or ultimate strengths for the alloy, as would be expected, the maximum von Mises stress (~370 MPa) was slightly less than the yield stress due to the hydrostatic stress component.

As the load was reduced from the overload to the baseline mean stress, $K_{I_{\text{Mean}}}$, it is clear that the field deviates significantly from a simple ‘$K_I$’ field. This is easily discerned from the compressive peak stress ($\sigma_{y_t}$) at, or just behind, the crack-tip in Fig. 6b. This is in good agreement with the behaviour sketched out in Fig. 1 and is more easily visualised in the line graphs of Fig. 7. When considering this plot it should be noted that the far field stresses under $K_{I_{\text{Min}}}$ in both directions are slightly compressive, as is the crack-opening stress at $2K_{I_{\text{Max}}}$ on the crack faces, suggest-
ing that the estimated stress free lattice parameter is probably slightly too large and that the stresses may be ~50 MPa too compressive. Provided there had been no movement of the sample during unloading, Fig. 7 confirms the presence of a significantly enhanced reverse plastic zone ahead of the crack, as well as a small region of crack face contact behind the crack-tip, even at $K_{\text{IMean}}$. Unfortunately, beam time constraints meant that it was not possible to measure the evolution of the stress field over many load steps during the unloading cycle and so it is not clear at what applied stress a compressive stress in the crack-tip region began to develop, but it would appear to be at a stress considerably larger than the baseline mean stress $K_{\text{IMean}}$ (in simple terms $K_{\text{IClosure}} > K_{\text{IMean}}$). Due to plastic stretch within the plastic zone the maximum tensile stress appears to be pushed some distance ($\sim 100$ μm) ahead of the crack-tip (cf. Fig. 1). This is slightly larger than the predicted extent of the overload forward plastic zone ($r_{\text{Oy}} \sim 60$ μm) and considerably larger than the estimated reverse plastic zone. The maximum tensile stress continues to fall as the load is removed such that it is relatively small at $K_{\text{IMin}}$. It would appear that the compressive zone behind the tensile peak due to the combined action of the plastic wake and the reversed plastic zone (Fig. 1) fails to maintain a significant tensile crack-tip stress intensity (even given the likely 50 MPa underestimation of the tensile stress), as the peak tensile stress was not that much different to that at $K_{\text{IMean}}$ for S1 measured during baseline fatigue. The tensile peak fell further on unloading to $K_{\text{IMin}}$. There is some evidence that the stress ahead of the reverse plastic zone is actually tensile (50 MPa) rather than zero, as it appears to be in Fig. 7, in view of the difficulty of accurately measuring the unstrained lattice parameter. The form of the stress field was broadly in agreement with Dugdale-type calculations, as well as the finite element calculations reported by Wang et al. [45]. Furthermore, our overload residual stress results in Fig. 6c are broadly similar to their predictions, taking into account growth of the fatigue crack, summarised in Fig. 13 of their paper. Their work suggested that the reverse plastic zone is more important than crack face closure in influencing crack growth. Recent work by Croft et al. [20] for thinner samples (more prone to

Fig. 7. Variation in (a) crack growth stress ($\sigma_{xx}$) and (b) crack-opening stress ($\sigma_{yy}$) along the crack plane (at $y = 0$) around the crack-tip of the FO specimen (S2) for $2K_{\text{IMax}} = 13.2$ MPa m, $K_{\text{IMean}} = 3.6$ MPa m and $K_{\text{IMin}} = 0.6$ MPa m. It is possible that overall the measured stresses are ~50 MPa too compressive (see text).
crack-tip plasticity) is in striking agreement with the results presented here. They observed a similar triangular profile in the region of the crack-tip at $K_{\text{Min}}$ and a compressive lobe constrained by a tensile region ahead of the crack-tip and a more striking one than we observed behind it. Unfortunately, they did not monitor the unloading response after overload, but they did monitor the loading response for the crack-tip stress file once it had propagated further; which shows striking similarities to the present work.

One way of examining the extent of the plastic zone, as well as the plastic wake, is to examine the diffraction peak widths. Peak width is usually very sensitive to line broadening by plastic work, although it can also be affected by any sharp gradient of elastic strains present within the gauge (as might occur around a crack-tip). The diffraction peak widths of the (single peak) fitted Al (311) reflection at $2K_{\text{Max}}$ for S2 and $K_{\text{Max}}$ for S1 are compared in Fig. 8. In agreement with the earlier estimation of the likely plastic zone size, it is clear that it was not possible using a 25 μm

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**Fig. 9.** Stress maps around the crack-tip of the FOF specimen (S3) at $K_{\text{Min}} = 0.6 \text{ MPa} \sqrt{\text{m}}$ (LHS) and $K_{\text{Max}} = 6.6 \text{ MPa} \sqrt{\text{m}}$ (RHS) for the (a) crack growth ($\sigma_{xx}$), (b) crack opening direction ($\sigma_{yy}$) and (c) normal stress ($\sigma_{zz}$) directions.
gauge dimension to resolve the plastic zone at $K_{\text{Max}}$. Nor was there any significant evidence of a plastic zone generated in the crack wake as the crack had grown. In contrast, broadening of the diffraction peak local to the crack-tip for the overloaded sample (S2) is clearly evident in Fig. 8b, despite the fact that energy dispersive diffraction typically has moderate instrumental resolution (peak width) compared with angular dispersive measurements [46]. This broadening may in part be a consequence of the steep elastic gradients found there (see also the discussion relating to Fig. 11), but it is likely that plasticity has also paid a role. The peak in the diffraction peak width profile appears to have an extent of around 200 $\mu$m – considerably larger than the predicted plastic zone ($\sim 45 \mu$m), but comparable with the reversed plastic zone inferred from Fig. 7 by comparing it with the schematic in Fig. 1.

3.3. Baseline fatigue cycle sometime after overload (S3)

Croft et al. [20] found that the effect of the overloaded plastic zone (which in their case was much larger) was retained as the crack was grown through the retardation zone and concluded that this was important in terms of retarding crack growth rate. In our experiment we examined the crack-tip stress field after the crack had extended a further 0.1 mm beyond the 100% ($2K_{\text{Max}}$) overload, i.e. just beyond the plastic zone arising from the overload.

The crack-tip stress fields after the crack had grown 0.1 mm beyond the 100% overload are shown in Fig. 9.

Comparing the stress fields at $K_{\text{Max}}$ between S1 (fatigued only) and S3 (FOF) in Figs. 4 and 9, respectively, the stress fields appear to be similar before and after the overload cycle in both cases. Indeed, the crack-tip stress intensity inferred from the full field strain map was 6.2 MPa$\sqrt{\text{m}}$ (Table 1), which is in good correspondence with that nominally applied, as well as with that observed at the crack-tip for baseline fatigue. This high level of correspondence was emphasised by direct comparison of the two crack-tip plane profiles in Fig. 10a and b, where the profiles have been displaced relative to one another to bring the crack locations into coincidence. The crack-opening stress at $K_{\text{Mean}}$ for the FO (S2) and FOF (S3) cases are shown overlaid in Fig. 10c. The effect of the overload cycle was marked immediately after unloading (FO). In contrast to Croft et al. [20], who observed a crack in a thinner sample, we did not see a significant change in the baseline fatigue residual stress field soon after overloading, nor any residual evidence of the overload feature. This may be because the plastic event created by the overload in our plane strain case was very small. Results at $K_{\text{mean}}$ for the FOF specimen S3 (Fig. 10c and d) look very similar to that for baseline fatigue, suggesting that the overload had not influenced the crack-tip stress fields, at least above the nominal $K_{\text{mean}}$ loading level. Furthermore, there appears no evidence that there was any residual effect 0.1 mm behind the crack-tip in the stress maps of Fig. 9.

To see if there was any evidence of the overload event, the diffraction peak width maps for sample S3 (FOF) are...
Fig. 11. The width of the Al (3 1 1) diffraction peak (in channel numbers) for S3 (FOF) for: (a) $K_{I_{\text{Max}}} = 6.6 \text{ MPa} \sqrt{\text{m}}$, (b) $K_{I_{\text{Mean}}} = 3.6 \text{ MPa} \sqrt{\text{m}}$ and (c) $K_{I_{\text{Min}}} = 0.6 \text{ MPa} \sqrt{\text{m}}$. The overload was applied when the crack was 100 \( \mu \text{m} \) behind the current location of the crack.

shown for $K_{I_{\text{Max}}}$, $K_{I_{\text{Mean}}}$ and $K_{I_{\text{Min}}}$ in Fig. 11. A broadening of the peak width is clear at the crack-tip location at all three load levels, but it is less than observed for the overload case shown in Fig. 8. Further, the peak width at the crack-tip decreased with unloading, perhaps indicating that its origin lay in the steep stress gradients there. In any event, evidence of the overload event as well as other crack wake peak broadening become increasingly evident with decreasing load. In particular, at $K_{I_{\text{Min}}}$ broadening became significant at \( \sim 100 \mu \text{m} \) behind the crack-tip (i.e. at the overload location). Further, there is some indication of some residual broadening event 0.65 mm behind the crack-tip, perhaps suggestive of crack face contact there to.

4. Conclusions

This paper presents two-dimensional maps of the elastic crack-opening and crack growth direction strains in the vicinity of a fatigue crack under plane strain conditions at much higher spatial resolution than has been achieved before [17]. From these measurements it has been possible to map the crack opening, transverse and normal stresses using an assumption of plane strain. Furthermore, because a large part of the diffraction profile had been recorded at each point, it was possible to use a Rietveld-style refinement so that the elastic strains determined are likely to be representative of bulk elastic strains/stresses. Additionally, by comparing the measured strain field with that predicted by theory it has been possible, for the first time, to infer the crack-tip stress intensity right at the crack-tip within the bulk under conditions of plane strain.

During baseline fatigue ($K_{I_{\text{Max}}} = 6.6 \text{ MPa} \sqrt{\text{m}}$) the elastic strain field local to the crack was indicative of a stress intensity very close to that nominally applied recording a peak tensile stress (averaged over the 25 \( \mu \text{m} \) gauge volume) of \( \sim 450 \text{ MPa} \). Upon unloading, however, no significant
residual stress was found at $K_{\text{IMin}}$, perhaps because of the difficulty of closure under plane strain or, alternatively, because the expected reverse plastic strain/zone size was very small and therefore their effects were not detected.

As at $K_{\text{IMax}}$, the strain field local to the crack-tip at 100% overload was consistent with the nominally applied stress intensity ($\approx13.2$ MPa$\sqrt{\text{m}}$), i.e. the crack-tip was loaded to the nominal stress intensity. During unloading compressive residual stresses were measured just in front of the crack indicative of a significant reverse plastic zone, as well as some evidence of a small region of crack closure behind the crack-tip due to crack face contact. The broad profile of these compressive strains compared well with those observed by Croft et al. [20] on thinner samples. The peak compressive stress was around $-450$ MPa. Slightly further ahead of the crack (around 100 $\mu$m) tensile stresses were evident at $K_{\text{IMin}}$, being maintained at no more than 100 MPa. It was not possible to infer from our unloading experiments the load at which the crack-tip stress became compressive, but significant compressive stresses were present even at $K_{\text{IMean}}$ due largely to reverse plastic yielding. In this context it should be noted that our residual stress curves were similar in form to the surface stress study of overload made by Jagg and Scholtes [47], except that the effects occurred on a finer scale. This is, of course, in part due to the fact that they studied plane stress, where plasticity-induced closure is much more dramatic and partly because they applied a higher maximum stress intensity. After crack growth under baseline fatigue to 0.1 mm, beyond the overload event, the stress intensity at $K_{\text{IMax}}$ was indistinguishable from that prior to the overload event.

In all cases the stress in the wake of the crack appeared to be compressive – this may be a valid observation or it may reflect an incorrect choice of the strain free lattice parameter. Croft et al. [18] found a similar effect using a single peak analysis. We tend to agree with the authors that in their case this was due to anisotropy arising from intergranular strains created by differential plastic flow between differently oriented grains. It is rather more difficult to explain in our case, because a Rietveld multiple peak analysis was used. If the effect is due to an error in the strain free lattice spacing then the stresses reported here are $\approx50$ MPa too compressive. Others have seen similar effects, if at lower spatial resolution [48]. Further work needs to be done both immediately after overload and after continued growth of the crack as it advances through the overload plastic zone to establish whether crack closure or other influences arising from the overload event can explain fatigue crack retardation under plane strain conditions, requiring significant amounts of beam time. A great deal has already been learnt from the study of Croft et al. [18,20] on thinner (plane stress) samples.

Finally, it should be noted that in our case the forward and reverse plastic zones were very small and difficult to detect, even using the excellent spatial resolution achieved here (25 $\mu$m). It would certainly be easier to confirm the presence or absence of plasticity-induced closure in systems with a larger plastic zone. This will be the target of further work carried out deep within thick plane strain samples in order to assess whether plasticity-induced closure events can be significant under plane strain conditions.

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